2. Vanishing derivatives for local extrema.

local extrema at x_0 $\xrightarrow{\text{necessary}}$ $f'(x_0) = 0$

local extrema at x_0 $\overset{\times}{\longleftarrow}$ (sufficient) $f'(x_0) = 0$

3. minimum, maximum, inflection pt.

$$f'(x_0) = f''(x_0) = \cdots = f^{(n-1)}(x_0) = 0, \quad f^{(n)}(x_0) \neq 0.$$

1. n is even: extrema
   - $f^{(n)}(x_0) > 0 \to$ min.
   - $f^{(n)}(x_0) < 0 \to$ max.
2. n is odd: inflection pt.

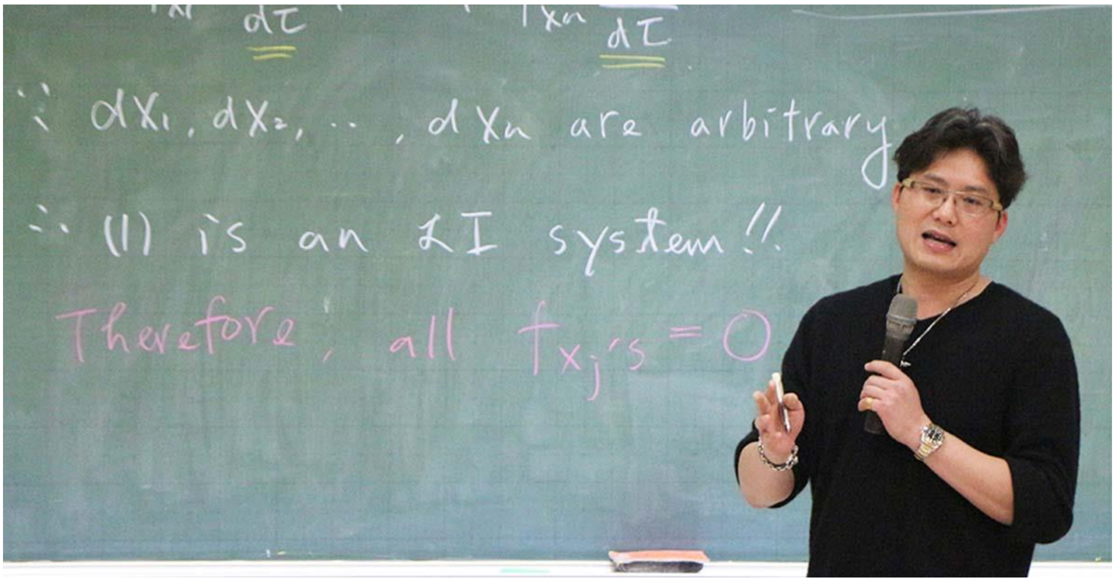
II. Multivariable cases.

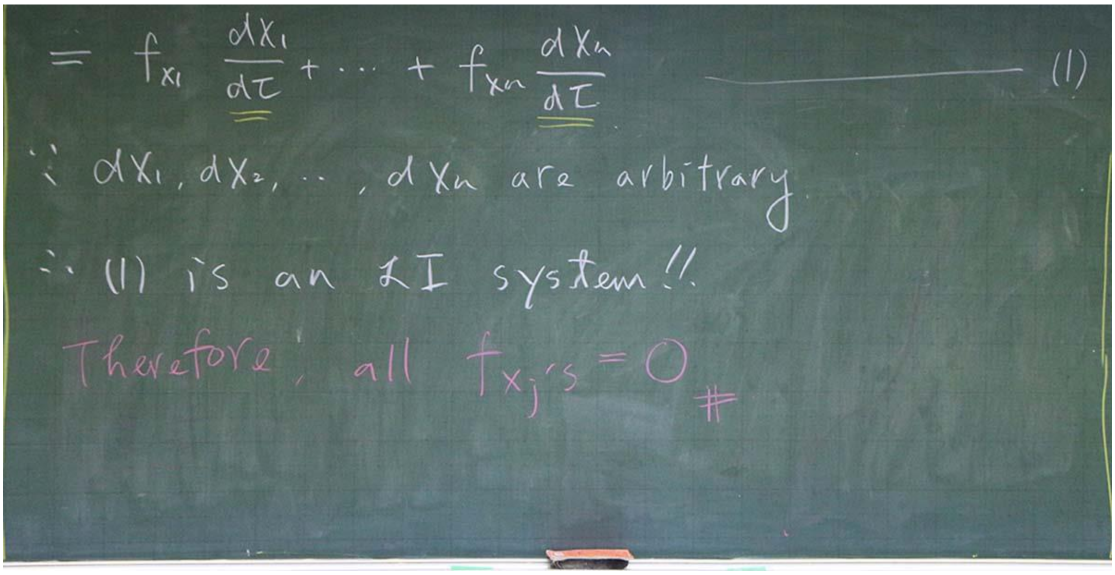
1. Vanishing derivatives: all f_{x_j}'s $= 0$.

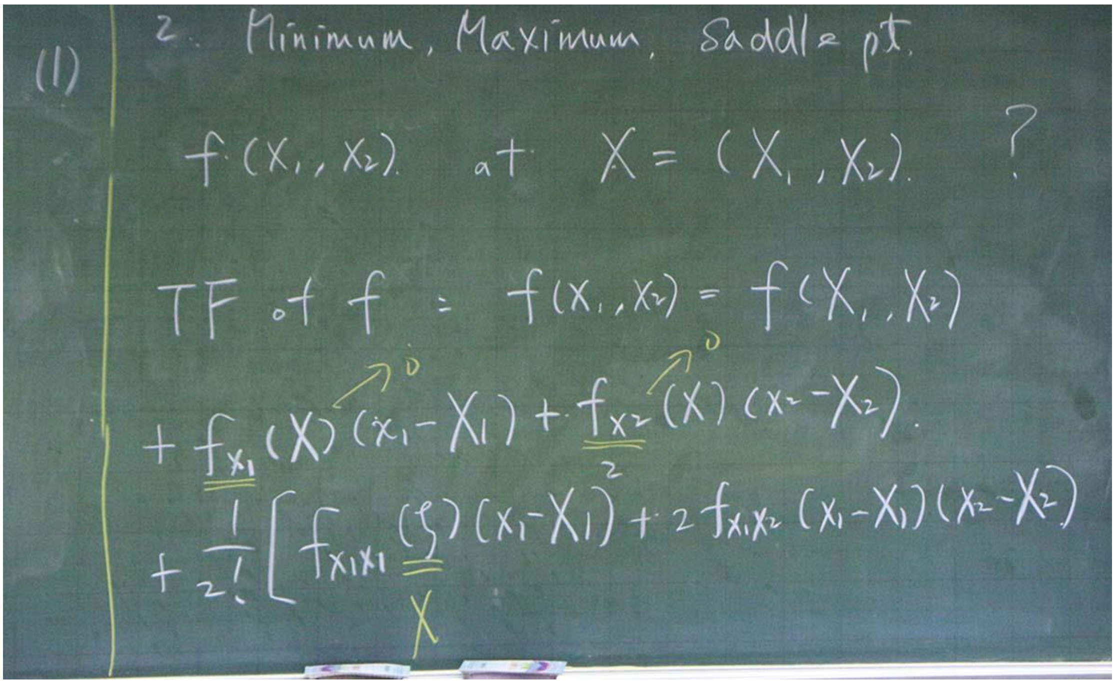
$$f \equiv f(x_1, \cdots, x_n) \; ; \quad x_1 \equiv x_1(\tau),\ x_2(\tau),\ \cdots,\ x_n(\tau)$$

$$\therefore f(x_1, \cdots, x_n) = f(x_1(\tau), x_2(\tau), \cdots, x_n(\tau))$$

$$\equiv F(\tau) \; ; \quad \frac{dF(\tau)}{d\tau} = 0 = \frac{\partial f}{\partial x_1}\frac{dx_1}{d\tau} + \cdots + \frac{\partial f}{\partial x_n}\frac{dx_n}{d\tau}$$


$\frac{dX_1}{d\tau} \cdots f_{x_n} \frac{dX_n}{d\tau}$
∵ $dX_1, dX_2, \ldots, dX_n$ are arbitrary
∴ (1) is an LI system!!
Therefore, all f_{x_j}'s = 0


$= f_{x_1} \frac{dX_1}{d\tau} + \cdots + f_{x_n} \frac{dX_n}{d\tau}$ ——— (1)
∵ $dX_1, dX_2, \ldots, dX_n$ are arbitrary
∴ (1) is an LI system!!
Therefore, all f_{x_j}'s = 0 #


(1)
2. Minimum, Maximum, Saddle pt.
$f(X_1, X_2)$ at $X = (X_1, X_2)$?
TF of f : $f(x_1, x_2) = f(X_1, X_2)$
$+ f_{x_1}(X)(x_1 - X_1) + f_{x_2}(X)(x_2 - X_2)$
$+ \frac{1}{2!}\left[f_{x_1x_1}(\xi)(x_1 - X_1)^2 + 2 f_{x_1x_2}(x_1 - X_1)(x_2 - X_2) \right.$
X

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(d)
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→ Wed.
(3/17).
13-14.

$$+ f_{x_2x_2}(\xi)\,(x_2-X_2)^2 \Big] \quad \text{———— (2)}$$

$\because f_{x_1} = f_{x_2} = 0$; $\underline{(x_1, x_2) \longrightarrow (X_1, X_2)}$

(2) → $f(x_1, x_2) - f(X_1, X_2)$

$$= \frac{1}{2}\Big[f_{x_1x_1}(X)\,(x_1-X_1)^2 + 2 f_{x_1x_2}(x_1-X_1)(x_2-X_2) + f_{x_2x_2}(x_2-X_2) \Big] \quad \text{—— (3)}$$

(X ↑ ξ)

(3) is a typical "quadratic form"

$$\text{RHS} = \frac{1}{2}\, Y^T_{1\times 2}\, A_{2\times 2}\, Y_{2\times 1} \quad \text{—— (4)}$$

$Y^T \downarrow \left[(x_1-X_1), (x_2-X_2) \right]$

$Y \downarrow \begin{bmatrix} (x_1-X_1) \\ (x_2-X_2) \end{bmatrix}$

$$A = \begin{bmatrix} f_{x_1x_1} & f_{x_1x_2} \\ f_{x_1x_2} & f_{x_2x_2} \end{bmatrix}$$

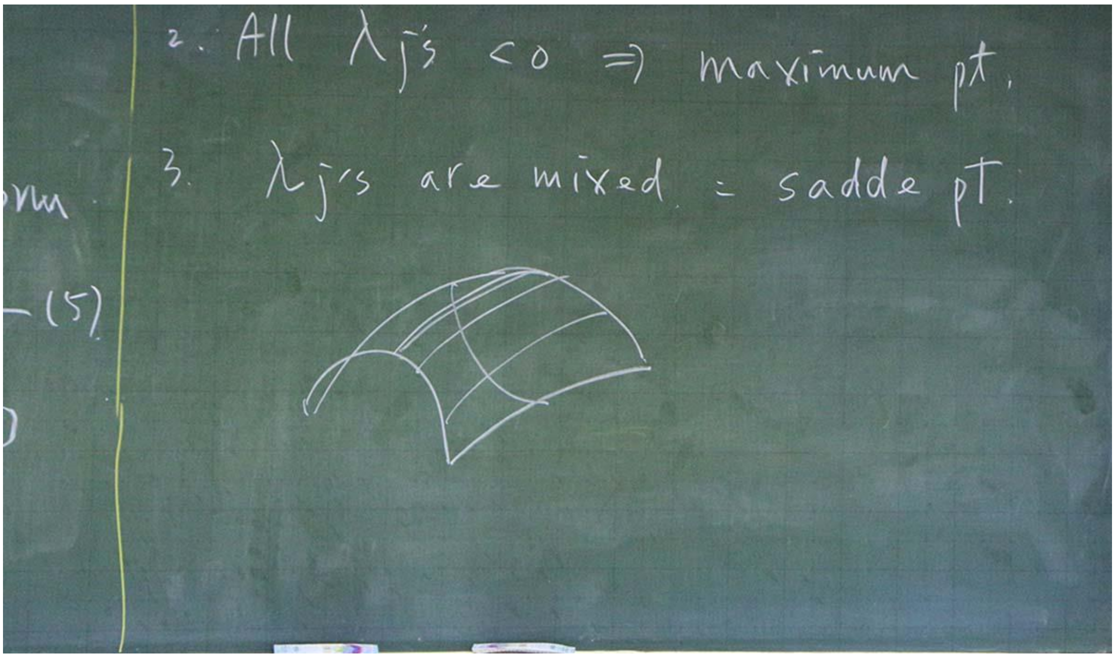
$\because$ A is a symmetric matrix

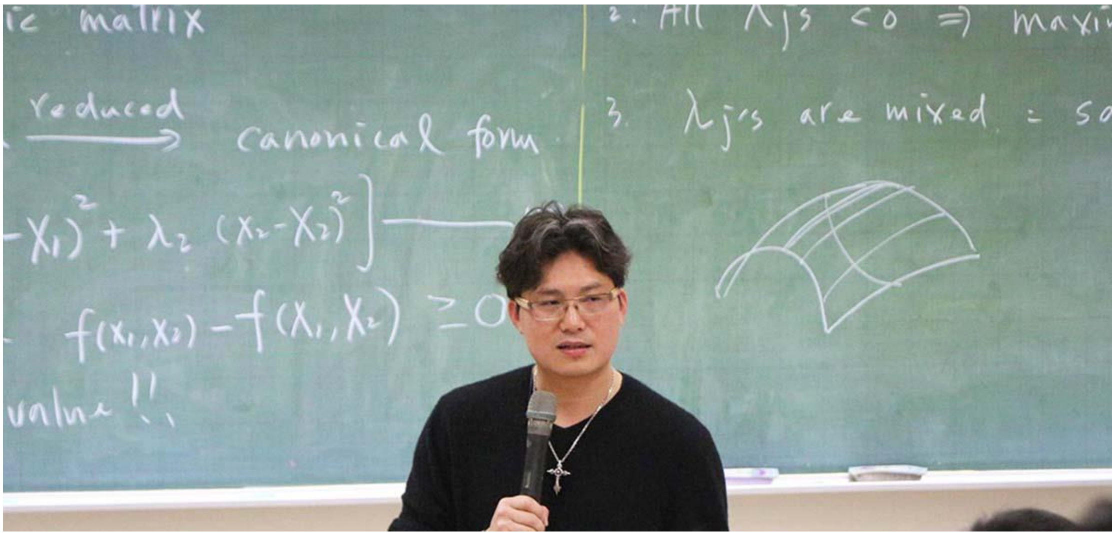
$\therefore$ quadratic form $\xrightarrow{\text{reduced}}$ canonical form

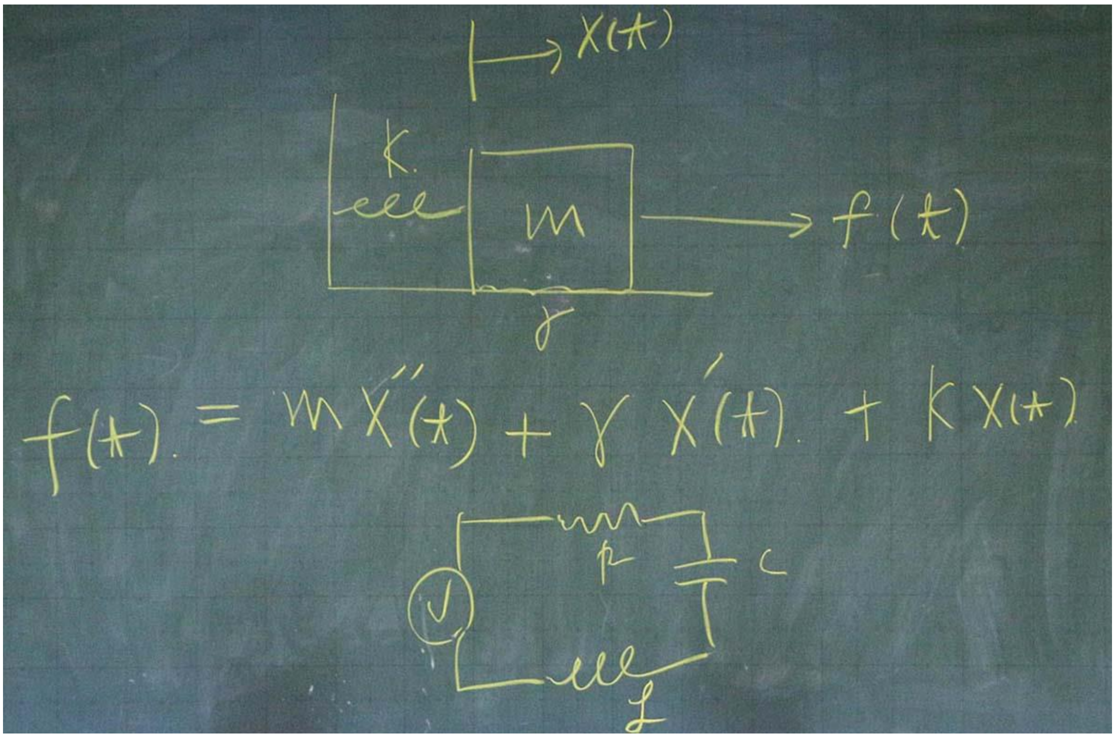
$$\text{RHS} = \frac{1}{2}\Big[\lambda_1 (x_1-X_1)^2 + \lambda_2 (x_2-X_2)^2 \Big] \quad \text{———— (5)}$$

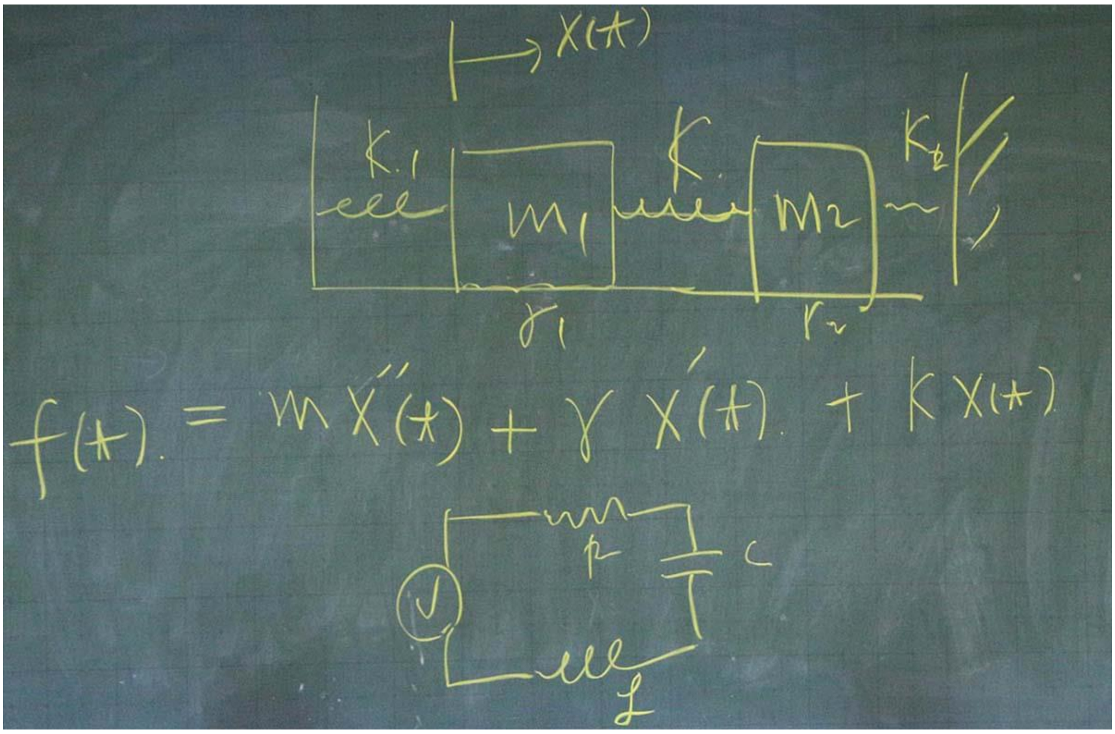
1. All λ_j's > 0 $\therefore$ $f(x_1, x_2) - f(X_1, X_2) \geq 0$

minimum value!!


2. All λj's < 0 ⇒ maximum pt.
3. λj's are mixed = sadde pt.
(5)


matrix
reduced
canonical form
2. All λj's < 0 ⇒ maxi
3. λj's are mixed = sa
f(x1,x2) − f(X1,X2) ≥ 0
value !!


x(t)
k
m
f(t)
γ
f(t) = m x''(t) + γ x'(t) + k x(t)
R
C
V
L


x(t)
k1
k
k2
m1
m2
γ1
γ2
f(t) = m x''(t) + γ x'(t) + k x(t)
R
C
V
L